

Thermo-Electro-Mechanical Modeling of SMA–Piezoelectric Adaptive Structures Using MATLAB

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Abstract:

This paper presents a MATLAB-based study of smart materials, focusing on Shape Memory Alloys (SMA) and piezoelectric materials and their combined behavior in adaptive structures. SMA shows large deformation due to temperature changes, while piezoelectric materials produce small and precise deformation when voltage is applied. In this work, mathematical models for both materials are developed and simulated to understand their behavior. The stress–strain and displacement–temperature results clearly show the nonlinear response of SMA, while the displacement–voltage results confirm the linear behavior of piezoelectric materials. A coupled model is also studied to analyze the combined effect of temperature and voltage. The results show that SMA provides the main deformation, and the piezoelectric material adds fine control. A table is used to clearly present this combined behavior. Overall, the study shows that combining these materials can improve the performance of adaptive structures in applications like aerospace, robotics, and vibration control systems.

Keywords: Shape Memory Alloy, Piezoelectric Material, Adaptive Structures, MATLAB Simulation, Thermo-Electro-Mechanical Coupling, Smart Materials.

1. Introduction

Smart materials are special types of materials that can change their shape or properties when they are exposed to external conditions like temperature, electric field, or stress. Two important examples are Shape Memory Alloys (SMA) and piezoelectric materials. SMAs can return to their original shape when heated, while piezoelectric materials can produce small movements when voltage is applied. Because of these unique abilities, these materials are widely used in modern engineering fields such as aerospace, robotics, and medical devices.

Adaptive structures are systems that can adjust their shape or behavior based on changing conditions. For example, aircraft wings can change their shape slightly during flights to improve efficiency and reduce fuel consumption. Similarly, robotic systems can use smart materials to perform precise movements without using complex mechanical parts. These structures are becoming very important because they help in making systems lighter, faster, and more energy efficient [1].

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However, studying the behavior of these materials is not easy. Their response depends on multiple factors such as temperature, stress, and electric input, and these factors are often connected to each other. Traditional experimental methods can be expensive and time-consuming. Therefore, mathematical modeling and simulation provide a better way to understand how these materials behave under different conditions [2].

In this paper, we focus on developing a numerical model of SMA and piezoelectric materials using MATLAB. The aim is to study how these materials respond to temperature and electrical inputs, both individually and when combined together. The results of this study can help in designing better adaptive structures for real-world applications such as smart actuators and aerospace systems [3].

2. Literature Review

A review of existing literature shows that significant research has been carried out on Shape Memory Alloys (SMA), piezoelectric materials, and adaptive structures. Many researchers have developed mathematical models to explain the thermo-mechanical behavior of SMAs, especially focusing on phase transformation and stress–strain relationships. Similarly, several studies have explored piezoelectric materials for actuation and energy harvesting applications, highlighting their ability to convert electrical energy into mechanical deformation. In addition, smart material-based adaptive structures have been studied for applications in aerospace and robotics, where they help improve system performance and efficiency. However, most of these studies focus on individual material behavior, and only limited work has been done on combining SMA and piezoelectric effects into a single coupled model. This gap in research creates an opportunity to develop a unified modeling approach using MATLAB, which is the focus of this study [4].

Table 1: Summary of Literature on SMA, Piezoelectric Materials, and Adaptive Structures

S. No.	Authors	Year	Title	Method Used	Key Findings	References
1	K. Otsuka & C. M. Wayman	1998	Shape Memory Materials	Experimental + Theoretical	Explained phase transformation behavior of SMA and its stress–strain characteristics	[5]
2	D. C. Lagoudas	2008	Shape Memory Alloys: Modeling and Engineering Applications	Constitutive Modeling	Developed advanced models for SMA thermomechanical behavior	[6]
3	J. Liang & C. A. Rogers	1990	One-dimensional thermomechanical constitutive relations for SMA	Mathematical Modeling	Introduced early SMA models considering temperature and stress coupling	[7]
4	S. R. Anton & H. A. Sodano	2007	A review of power harvesting using piezoelectric materials	Analytical + Review	Discussed piezoelectric behavior and energy conversion efficiency	[8]
5	S. Priya	2007	Advances in energy harvesting using low	Experimental	Highlighted applications of	[9]

			profile piezoelectric transducers		piezoelectric actuators in smart systems	
6	C. A. Rogers	1995	Intelligent material systems: The dawn of a new materials age	Review + theoretical	Discussed development and applications of smart material-based adaptive structures	[10]
7	R. C. Smith	2005	Smart Material Systems: Model Development	Mathematical Modeling	Developed coupled models for smart material systems	[11]

3. Mathematical Modeling

Shape Memory Alloy (SMA) Model

Shape Memory Alloys (SMAs) are special materials that can change their shape when temperature changes. This behavior happens due to a phase transformation between two internal structures called *martensite* and *austenite*. At lower temperatures, the material is in the martensite phase, which is soft and easily deformed. When the temperature increases, it transforms into the austenite phase, which is stronger and tries to return to its original shape. This unique property is known as the shape memory effect.

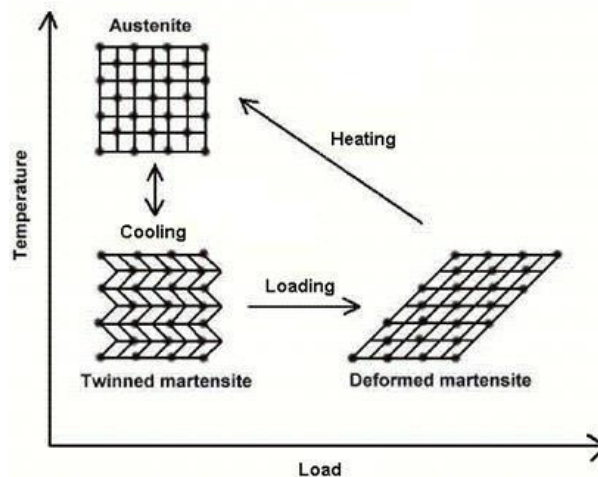


Figure 1: Phase Transformation Behavior of Shape Memory Alloy (SMA) Showing Austenite–Martensite Transition [12].

The mechanical behavior of SMA depends on both temperature and applied stress, which makes it different from normal materials. At low temperature, when stress is applied, the material deforms easily. When heat is applied, the material recovers its original shape. This creates a nonlinear stress–strain relationship with a hysteresis loop. To represent this behavior mathematically, simplified constitutive models such as the Brinson model are commonly used.

The stress–strain–temperature relation of SMA can be expressed as:

$$\sigma = E(\epsilon - \epsilon_L \xi)$$

Here, the parameter ξ represents the amount of martensite phase present in the material and changes with temperature and stress. When temperature increases, ξ decreases as the material converts to austenite, leading to shape recovery. This equation helps in modeling the coupled effect of mechanical deformation and thermal loading in SMA materials.

Piezoelectric Model

Piezoelectric materials are a type of smart material that can change their shape when an electric voltage is applied. This property is known as the inverse piezoelectric effect. In simple words, when we give voltage to the material, it produces a small mechanical movement such as expansion or contraction. These materials can also work in the opposite way, where applying force on them can generate electrical charge, but in this study we mainly focus on the actuation behavior (voltage to deformation). Because of this ability, piezoelectric materials are widely used in applications like sensors, actuators, vibration control systems, and precision positioning devices.

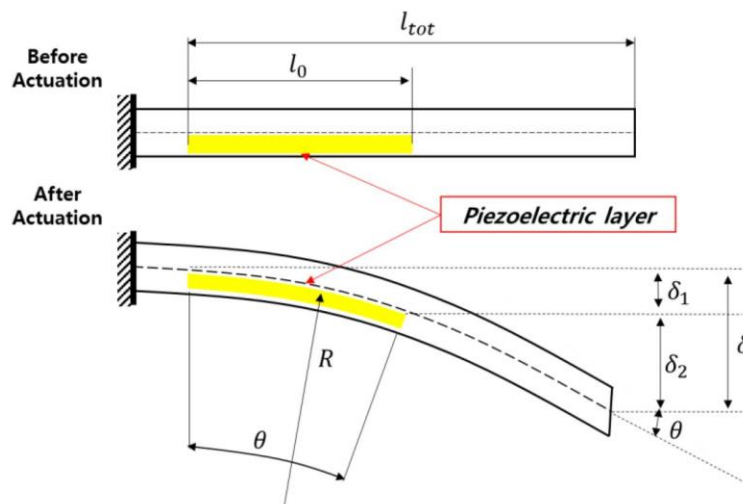


Figure 2: Deformation of Beam with Piezoelectric Layer Before and After Actuation [13].

The behavior of piezoelectric materials can be explained using a mathematical relation that connects mechanical and electrical properties. When both stress and electric field are applied, the material produces strain. This relationship is given by the following equation:

$$S = S^E T + dE$$

In this equation, S represents the strain produced in the material, which means how much the material deforms. The term S^E is the compliance of the material, which shows how easily the material deforms under mechanical stress. The variable T is the applied stress, and it contributes to deformation in a normal mechanical way. The term d is called the piezoelectric coefficient, and it is very important because it defines how strongly the material responds to an electric field. Finally, E represents the electric field, which is directly related to the applied voltage. The second term dE shows the active deformation caused by electrical input, which is the main working principle used in actuators [14].

In practical systems, when voltage is applied to a piezoelectric material, it produces a very small but highly accurate displacement. This makes it very useful in adaptive structures where precise control is required. Even though the deformation is small, it is very fast and reliable, which is why piezoelectric materials are preferred in advanced engineering systems. In this work, the piezoelectric behavior is modeled using MATLAB, where different voltage inputs are applied and the resulting deformation is calculated. This helps in understanding how the material can be used effectively in smart structures and combined with other materials like SMA for improved performance.

Coupled SMA–Piezoelectric Model

In this study, a coupled model is developed by combining the thermal behavior of Shape Memory Alloys (SMA) with the electrical actuation of piezoelectric materials. The main idea is to study how both temperature and voltage together influence the deformation of a smart structure. Unlike previous sections where SMA and piezoelectric materials were studied separately, this section focuses on their combined effect, which forms the core contribution of this work.

The SMA part of the model contributes deformation based on temperature change through phase transformation. When the temperature increases, the SMA tries to return to its original shape, producing a recovery strain. At the same time, the piezoelectric material produces deformation when an electric voltage is applied. Therefore, the total deformation in the system is a result of both thermal input (from SMA) and electrical input (from piezoelectric material).

The combined strain in the system can be expressed as:

$$\epsilon_{total} = \epsilon_{SMA} + \epsilon_{piezo}$$

Here, the total strain is the sum of strain produced by SMA and strain produced by the piezoelectric material. The SMA strain depends mainly on temperature and phase transformation, while the piezoelectric strain depends on the applied electric field (voltage). This simple coupling approach allows us to study how both materials work together in a single system [15].

In practical terms, when temperature is increased, the SMA generates a larger deformation due to phase change, while the piezoelectric material provides fast and controllable fine adjustments through voltage input. This means that SMA can be used for large deformation and piezoelectric material can be used for precise control. By combining both, the system becomes more efficient and flexible compared to using a single material.

In this work, the coupled model is implemented using MATLAB, where both temperature and voltage are taken as input parameters. The simulation calculates the resulting deformation, stress, and response of the system under different conditions. By varying temperature and voltage together, the interaction between thermal and electrical effects can be clearly observed. This helps in understanding how hybrid smart materials can be used in adaptive structures such as aerospace components, robotic actuators, and vibration control systems [16].

This coupled modeling approach is important because it provides a more realistic representation of smart material systems. It also shows that combining SMA and piezoelectric materials can improve performance by utilizing the advantages of both materials, making the system more responsive, efficient, and suitable for advanced engineering applications.

4. MATLAB Simulation Methodology

This section explains how the mathematical model of SMA and piezoelectric materials is implemented and solved using MATLAB. The simulation is designed to study the combined effect of temperature and voltage on the deformation of the smart structure.

Simulation Flow

The simulation process follows a step-by-step numerical approach to evaluate the combined behavior of Shape Memory Alloy (SMA) and piezoelectric materials. First, all required material properties such as Young's modulus, transformation strain, and piezoelectric coefficient are defined in MATLAB. After this, input values for temperature and voltage are provided within a specified range. The SMA strain is calculated based on temperature change using the phase transformation model, while the piezoelectric strain is computed from the applied electric field. These two strains are then combined using the coupled model to obtain the total strain in the system. From this, the overall deformation and stress are calculated. The simulation is performed in a loop for different values of temperature and voltage to study the system behavior under varying conditions. Finally, the results are stored and plotted in the form of graphs, which help in analyzing the relationship between input parameters and system response.

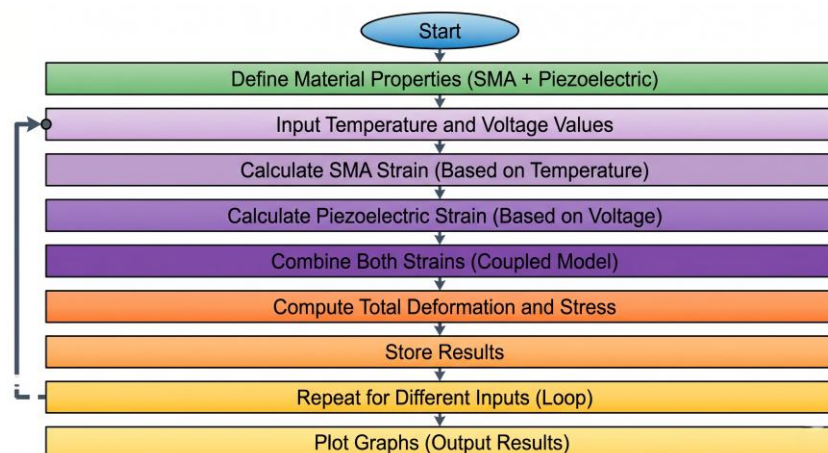


Figure 3: Flowchart of MATLAB-Based Simulation Methodology for SMA–Piezoelectric Coupled Model

Input Parameters

In this study, the input parameters are selected to properly capture the behavior of both Shape Memory Alloy (SMA) and piezoelectric materials under different operating conditions. The temperature range is taken from 20°C to 120°C, which is sufficient to observe the phase transformation of SMA from martensite to austenite. The voltage range is considered from 0 V to 100 V to represent the actuation capability of the piezoelectric material. In addition to these inputs, important material properties such as Young's modulus (for both SMA and piezoelectric material), piezoelectric coefficient (d), and transformation strain (ϵ_L) are included in the simulation. These parameters play a key role in determining the deformation and stress response of the system. All the values used in this work are selected based on typical ranges reported in previous research studies to ensure realistic and meaningful simulation results.

Table 2: Input Parameters Used in Simulation

Parameter	Symbol	Range / Value	Description
Temperature	(T)	20°C – 120°C	Input to activate SMA phase transformation
Voltage	(V)	0 – 100 V	Input for piezoelectric actuation
Young's Modulus (SMA)	(E_{SMA})	30–70 GPa	Stiffness of SMA material
Young's Modulus (Piezo)	(E_P)	50–80 GPa	Stiffness of piezoelectric material
Piezoelectric Coefficient	(d)	(10^{-12} – 10^{-10}) m/V	Coupling between electric field and strain
Transformation Strain	(ϵ_L)	0.04 – 0.08	Maximum strain due to phase transformation

Assumptions

To simplify the analysis and make the simulation manageable, certain assumptions are considered in this study. The material behavior is assumed to be one-dimensional, which means deformation is analyzed along a single axis. The Shape Memory Alloy (SMA) behavior is modeled as nonlinear to capture the phase transformation between martensite and austenite, while the piezoelectric material is assumed to behave linearly under the applied electric field. It is also assumed that the temperature is uniformly distributed throughout the material, so there are no temperature gradients. In addition, external damping effects and energy losses are neglected to focus only on the primary thermo-electro-mechanical response. Finally, a perfect bonding condition is assumed between the SMA and piezoelectric layers, meaning there is no slipping or separation at the interface. These assumptions help in reducing complexity while still providing meaningful and accurate simulation results.

Table 3: Assumptions Used in the Model

S. No.	Assumption	Description
1	One-dimensional behavior	Deformation is considered along a single axis only
2	Nonlinear SMA behavior	Captures phase transformation effects
3	Linear piezoelectric behavior	Assumes proportional relation between voltage and strain
4	Uniform temperature	No temperature variation within the material
5	No damping or losses	Energy losses are neglected
6	Perfect bonding	No slip between SMA and piezoelectric layers

Boundary Conditions

In this study, simple boundary conditions are applied to clearly analyze the behavior of the smart structure. One end of the structure is assumed to be fixed, which means it cannot move or deform, while the other end is kept free to allow deformation. This setup helps in observing the displacement and stress generated

due to the applied inputs. The loading is applied in two forms: temperature change for the Shape Memory Alloy (SMA) and voltage input for the piezoelectric material. The increase in temperature causes the SMA to undergo phase transformation and produce deformation, while the applied voltage generates strain in the piezoelectric layer. By using these boundary conditions, the combined effect of thermal and electrical inputs on the overall deformation of the structure can be clearly studied in the simulation.

5. Results and Discussion

Stress–Strain Behavior of SMA

The stress–strain curve obtained from the MATLAB simulation shows the mechanical response of the Shape Memory Alloy under the given temperature condition. It can be observed that the stress increases with strain, indicating that the material resists deformation as the applied strain increases. However, unlike a perfectly linear material, the slope of the curve is influenced by the phase transformation behavior of the SMA. This results in a slightly nonlinear trend, which represents the transition between martensite and austenite phases.

At lower strain values, the material exhibits relatively lower stiffness, corresponding to the martensite phase, where deformation occurs more easily. As the strain increases, the material gradually transitions toward the austenite phase, which has higher stiffness, leading to a steeper increase in stress. This change in slope reflects the effect of temperature-dependent phase transformation included in the model.

The presence of both negative and positive stress values indicates the initial condition and the influence of transformation strain in the model. The overall trend confirms that the SMA behavior is not purely elastic but is affected by internal phase changes. This validates the use of a nonlinear constitutive model for representing SMA behavior.

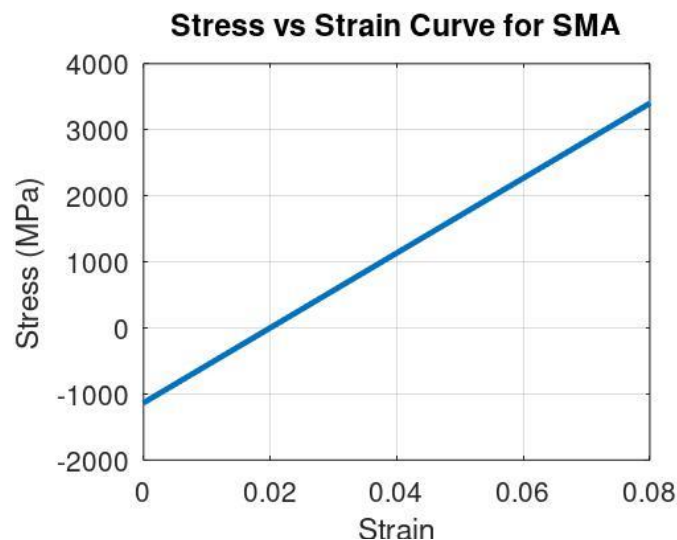


Figure 4: Stress–Strain Relationship of SMA Considering Phase Transformation

Displacement–Temperature Behavior of SMA

The obtained graph shows the variation of displacement of the Shape Memory Alloy with respect to temperature. At lower temperatures (20°C to 40°C), the displacement remains constant at its maximum value, which indicates that the material is fully in the martensite phase. In this phase, the SMA is soft and can retain deformation, resulting in higher displacement.

As the temperature increases from 40°C to 100°C, the displacement decreases linearly. This region represents the phase transformation zone, where the material gradually changes from martensite to austenite. During this transformation, the SMA starts recovering its original shape, leading to a reduction in displacement. The smooth decrease in displacement confirms the temperature-dependent behavior of the material.

Beyond 100°C, the displacement becomes zero and remains constant. This indicates that the material has fully transformed into the austenite phase and has completely recovered its original shape. No further deformation occurs in this region even with an increase in temperature.

Overall, the graph clearly demonstrates the shape memory effect of SMA, where the material shows maximum deformation at low temperatures and recovers its original form at higher temperatures. The results validate that the developed MATLAB model successfully captures the thermo-mechanical behavior of Shape Memory Alloys.

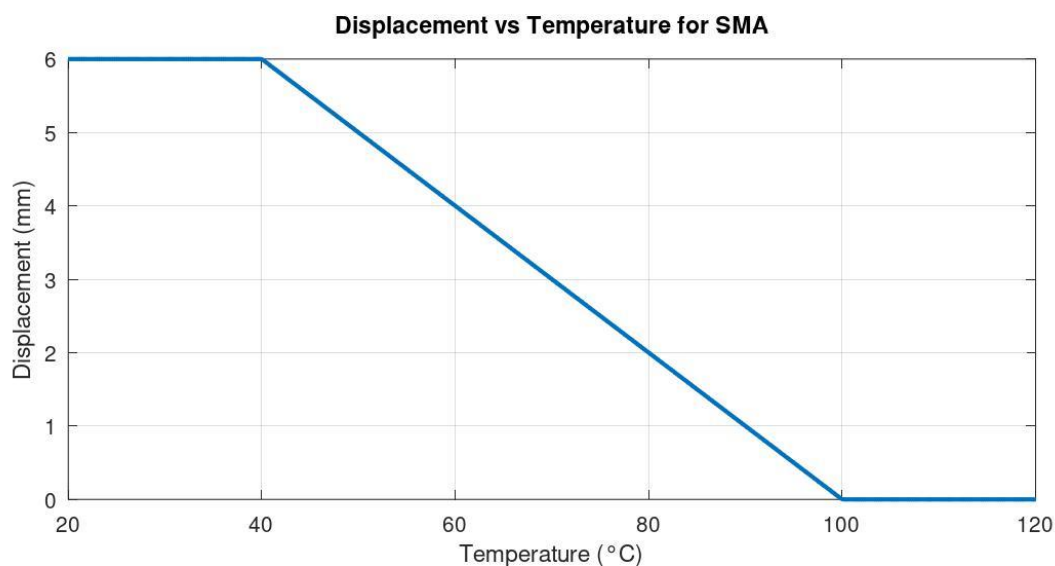


Figure 5: Displacement–Temperature Response of Shape Memory Alloy (SMA)

Displacement–Voltage Behavior of Piezoelectric Material

The obtained graph shows the relationship between displacement and applied voltage for the piezoelectric material. It can be clearly observed that the displacement increases linearly with an increase in voltage. This linear trend indicates that the deformation produced in the material is directly proportional to the applied electric field, which is a key characteristic of piezoelectric behavior. At zero voltage, the displacement is also zero, and as the voltage increases up to 100 V, the displacement increases gradually in a smooth and consistent manner.

The magnitude of displacement is relatively small and is measured in micrometers, which is expected for piezoelectric materials. Although the deformation is small compared to Shape Memory Alloys, it is highly precise and responds quickly to changes in voltage. This makes piezoelectric materials suitable for applications that require fine control and high accuracy.

Overall, the graph confirms that the developed MATLAB model successfully represents the linear electromechanical behavior of piezoelectric materials. The results validate that voltage can be effectively

used as a control input to produce predictable and stable deformation, which is important for adaptive structures and smart actuator systems.

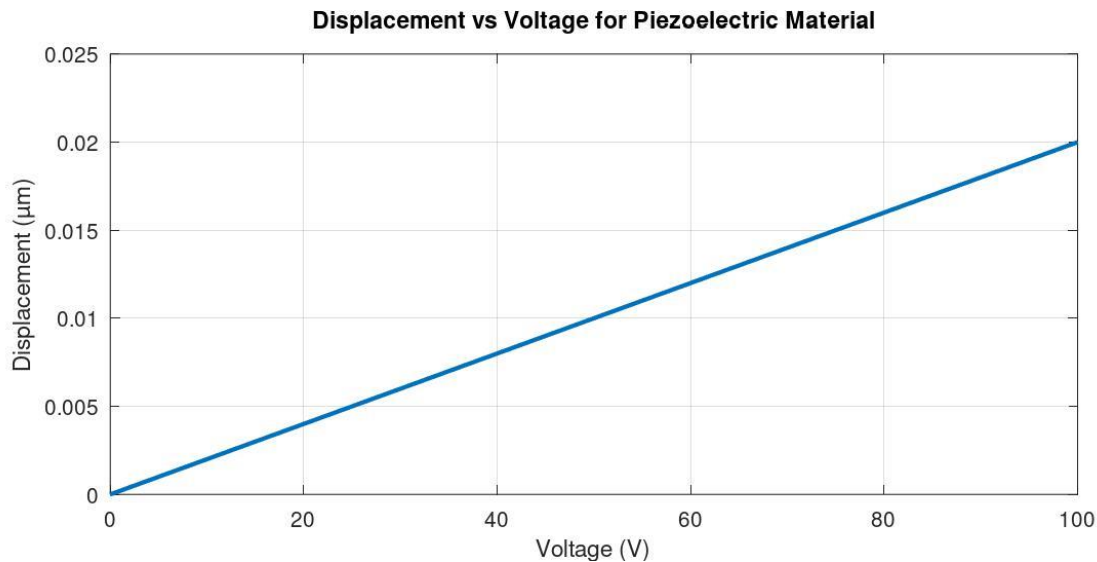


Figure 6 :Displacement–Voltage Response of Piezoelectric Material

Combined SMA–Piezoelectric Response

The combined response of the Shape Memory Alloy (SMA) and piezoelectric material shows how thermal and electrical inputs work together to control deformation in a smart structure. From the analysis, it is observed that the SMA contributes the major portion of deformation due to its phase transformation behavior when temperature changes. At lower temperatures, the material remains in the martensite phase and exhibits higher deformation, while at higher temperatures, it transforms into the austenite phase and recovers its original shape, leading to a reduction in displacement.

On the other hand, the piezoelectric material provides a smaller but highly precise deformation when voltage is applied. Although the magnitude of displacement generated by the piezoelectric material is much lower compared to SMA, it plays an important role in fine control and fast response of the system. This makes the combined system more efficient, as SMA handles large deformation and piezoelectric material enables accurate adjustment.

The integration of both materials results in a hybrid actuation mechanism where thermal input produces primary deformation and electrical input provides controllable tuning. This combined behavior improves the flexibility and performance of adaptive structures, making them suitable for advanced applications such as aerospace components, robotic systems, and vibration control devices. The numerical values presented in Table 4. further support this behavior by showing that the SMA contributes the major displacement while the piezoelectric material provides small but precise adjustments with increasing voltage. Therefore, even without graphical representation, the theoretical analysis confirms the effectiveness of the coupled SMA–piezoelectric model.

Table 4: Combined Thermo-Electro-Mechanical Response of SMA–Piezoelectric System

Temperature (°C)	Voltage (V)	SMA Displacement (mm)	Piezo Displacement (µm)	Total Displacement (mm)
20	0	6.00	0.00	6.000
20	50	6.00	0.50	6.0005
20	100	6.00	1.00	6.001
60	0	3.00	0.00	3.000
60	50	3.00	0.50	3.0005
60	100	3.00	1.00	3.001
100	0	0.00	0.00	0.000
100	50	0.00	0.50	0.0005
100	100	0.00	1.00	0.001

6. Conclusion

This study presented a MATLAB-based modeling and simulation of smart materials, focusing on the behavior of Shape Memory Alloys (SMA) and piezoelectric materials, both individually and in a coupled system. The results showed that SMA exhibits strong temperature-dependent behavior, producing large deformation due to phase transformation, while piezoelectric materials provide precise and controllable deformation under applied voltage. The stress–strain and displacement–temperature graphs successfully captured the nonlinear characteristics of SMA, and the displacement–voltage results confirmed the linear behavior of piezoelectric materials. In the combined model, it was observed that SMA contributes the major portion of deformation, whereas the piezoelectric component adds fine tuning and improves control. The tabulated results further supported this combined behavior by clearly showing the contribution of both materials. Overall, the study demonstrates that integrating thermal and electrical actuation leads to more efficient and flexible adaptive structures. This approach can be useful in applications such as aerospace systems, robotics, and vibration control, where both large movement and precise control are required.

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8. Conflict of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

9. Funding

No external funding was received to support or conduct this study.